Games with perfect information
Exercise sheet 5

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Out: May 15 Due: May 22

Submit your solutions on Wednesday, May 22, during the lecture.

Exercise 1: Parity games

a) Let $G = (V_\square \uplus V_\bigcirc, R)$ be a deadlock-free, finite game arena. Let $x, y \in V$ be two positions, $x \neq y$.

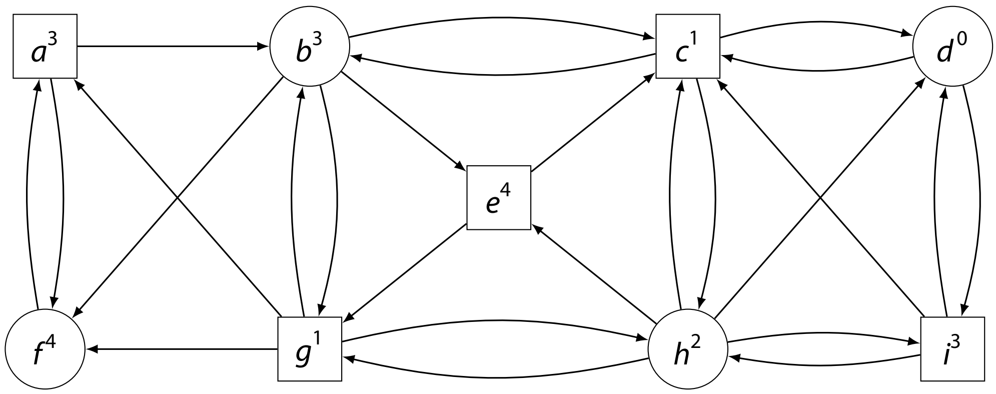
- Present a parity game whose winning condition encodes the following property:
  A play is won by the existential player if it either does not visit x infinitely often, or it visits both x and y infinitely often.

- Present a parity game whose winning condition encodes the following property:
  A play is won by the existential player if it visits x at least once, and later visits y infinitely often.

- Present a parity game whose winning condition encodes the following property:
  A play is won by the existential player if it either does not visit x infinitely often, or it visits x, but not y infinitely often.

Note: You are allowed to modify the game arena G if needed.

b) Consider the parity game given by the following graph. For each vertex labeled with x^i, the letter x denotes the name of the vertex, the superscript denotes its priority $\Omega(x) = i$.



For each player, identify her winning region and present a uniform positional winning strategy.

Note: We have not discussed the algorithm for solving parity games yet, but you should be able to do this!

Exercise 2: Uniform positional winning strategies

Prove Part b) of Lemma 6.5 from the lecture notes:

Let X be a set of positions such that for each $x \in X$, $\star \in \{\bigcirc, \square\}$ has a positional strategy $s_{\star,x}$ that is winning from x. Then there is a positional strategy $s_\star$ that is uniformly winning from all positions $x \in X$.

Hint: You can assume that $V = \{v_0, v_1, v_2, \ldots\}$ is countable. A proof by induction (using Part a) of Lemma 6.5) will not work, since X may be infinite. However, many of the arguments from the proof of Part a) of Lemma 6.5 can be reused.

Exercise 3: It's a trap!

a) Formally prove Part a) of Lemma 6.9 from the lecture notes:

   Let $Y \subseteq V$ and $\star \in \{\bigcirc, \square\}$. The complement of the attractor $V \setminus \mathrm{Attr}_\star(Y)$ is a trap for player $\star$.

b) Construct a game arena and a set Y such that $\mathrm{Attr}_\star(Y)$ is not a trap for any of the players. Proof that these properties hold.