On the Upward/Downward Closures of Petri Nets

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Goal

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Study the upward and downward closures of Petri net coverability languages

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Why?

Petri nets are an important model for concurrent systems

Upward and downward closures are useful approximations for verification purposes

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Upward/Downward closures in general

Good:

Always regular

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Always simply regular

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Upward/Downward closures in general

Good:

Always simply regular

Bad:

Representations might be not be effectively computable or very large

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Here:

- Closures effectively regular
- Want to construct finite state automata (FSA) as representations

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Here:

Closures effectively regular

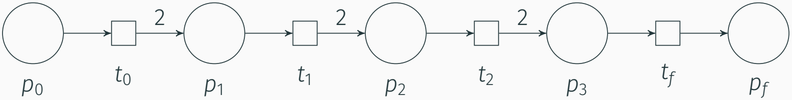
Want to construct finite state automata (FSA) as representations

↳ Time needed for the construction?

↳ Size of the minimal FSAs?

Petri Net Coverability Languages and their Closures

(Labeled) Petri Nets



$$N = \Big(\underbrace{\{p_0, p_1, p_2, p_3, p_f\}}_{\text{places } P}, \underbrace{\{t_0, t_1, t_2, t_f\}}_{\text{transitions } T}, F \Big)$$

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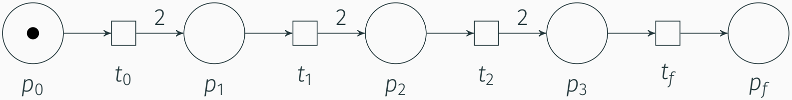

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$$\underset{M_0}{\underset{\parallel}{\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}}}$$

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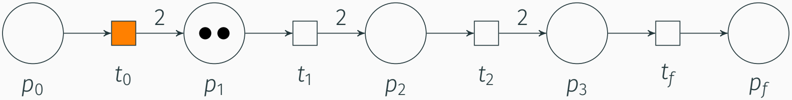

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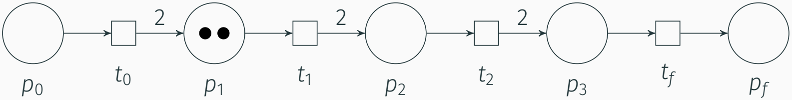

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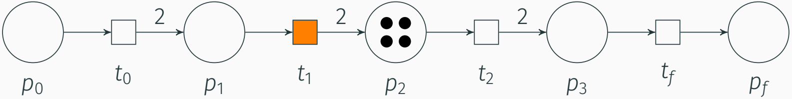

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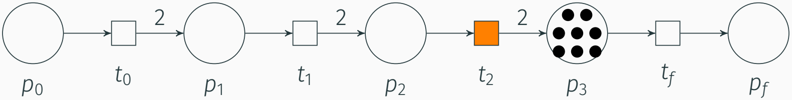

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$M_0 = (1, 0, 0, 0, 0)^T$, $M_f = (0, 0, 0, 0, 2^3)^T$

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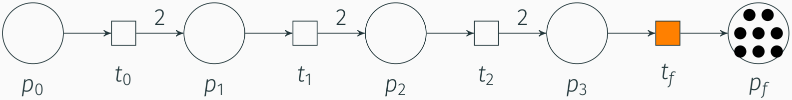

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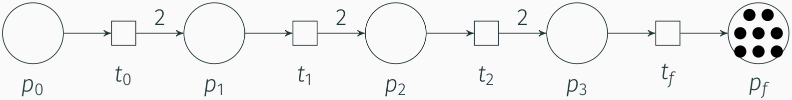

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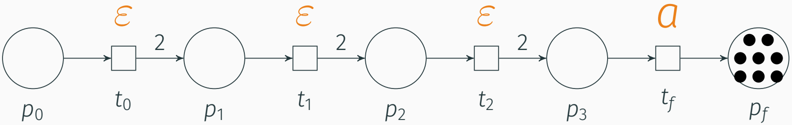

$$N = \left(\{a\}, \underbrace{\{p_0, p_1, p_2, p_3, p_f\}}_{\text{places } P}, \underbrace{\{t_0, t_1, t_2, t_f\}}_{\text{transitions } T}, F, \lambda\right)$$

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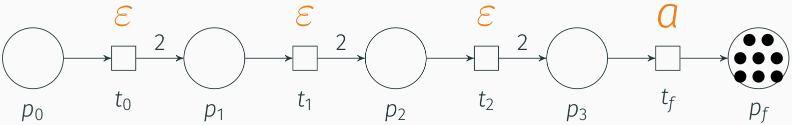

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$$\underset{M_0}{\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}} \ \underset{\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon\varepsilon aaaaaaaa = a^8}{\underset{\downarrow \lambda}{[t_0 t_1 t_1 t_2 t_2 t_2 t_2 t_f t_f t_f t_f t_f t_f t_f t_f\rangle}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 8 \end{pmatrix} \geqslant \underset{M_f}{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2^3 \end{pmatrix}}$$

Coverability Language

(N, M_0, M_f) Petri net with initial and final marking

Coverability language

$$\mathcal{L}\big(N, M_0, M_f\big) = \big\{\lambda(\sigma) \;\big|\; M_0[\sigma\rangle M, M \geqslant M_f\big\}$$

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In the example: $\mathcal{L}(N, M_0, M_f) = \{a^8\}$

Upward and Downward Closure

Subword relation

$$v \preceq w \quad \text{iff} \quad v \text{ obtained from } w \text{ by deleting letters}$$
$$\quad\quad\quad \text{iff} \quad w \text{ obtained from } v \text{ by inserting letters}$$

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In the example: $\mathcal{L}(N, M_0, M_f)\uparrow = \{a^k \mid k \geqslant 8\}$

$$\mathcal{L}(N, M_0, M_f)\downarrow = \{a^k \mid k \leqslant 8\}$$

Computing the Upward Closure

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Given: Petri net (N, M_0, M_f).

Compute: FSA A with $\mathcal{L}(A) = \mathcal{L}\big(N, M_0, M_f\big)\uparrow$.

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Theorem

Upper bound: *One can compute an FSA of doubly exponential size representing the upward closure in doubly exponential time.*

Lower bound: *This is optimal.*

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Lemma (Upper Bound)

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Petri net coverability can be solved using exponential space (and doubly exponential time).

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Show $f(i+1) \leqslant (2^n f(i))^{i+1} + f(i)$

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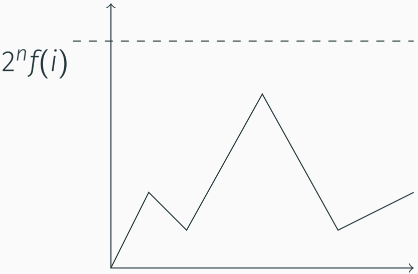
1st case: Values on all places $[1..i+1]$ bounded by $2^n \cdot f(i)$:



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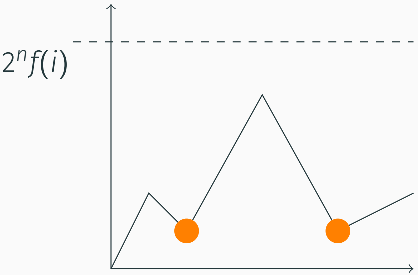

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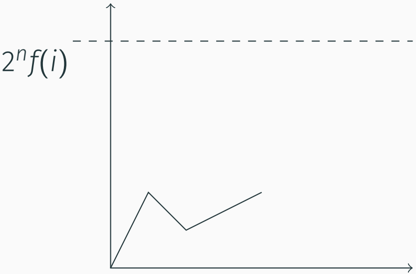

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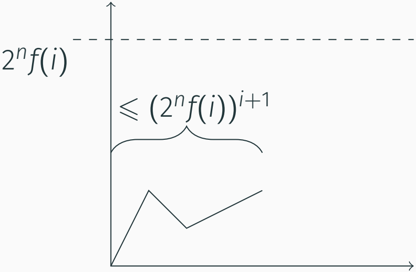
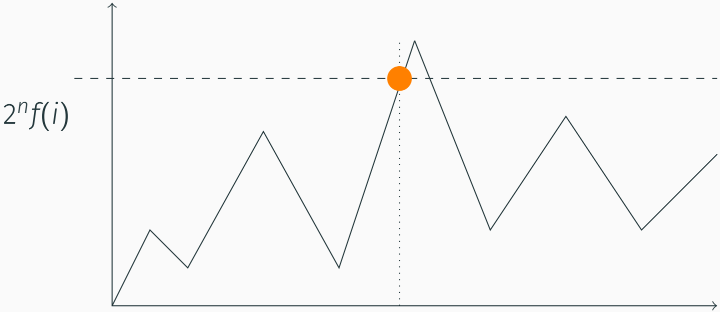
↳ Obtain new computation of length at most $(2^n f(i))^{i+1}$



Computing the Upward Closure - Upper Bound

2^{nd} case: Some place, say $i+1$, exceeds $2^n \cdot f(i)$:



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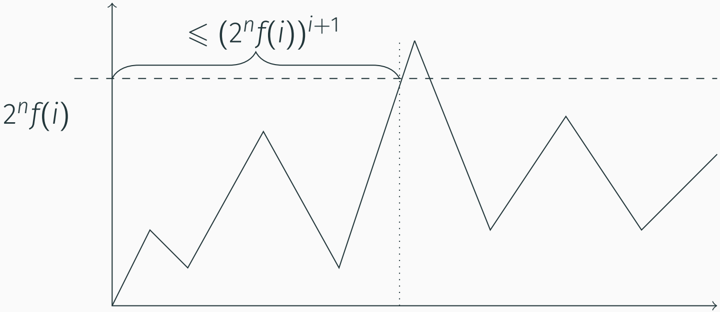

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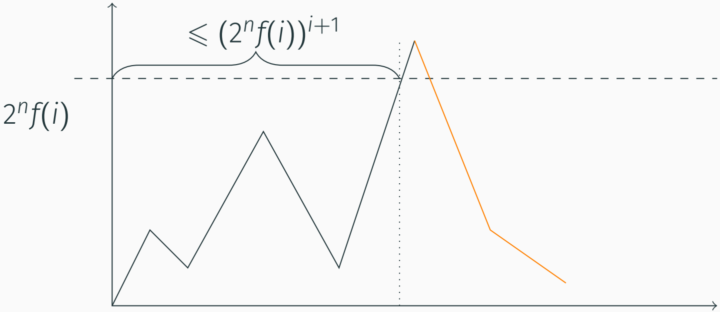

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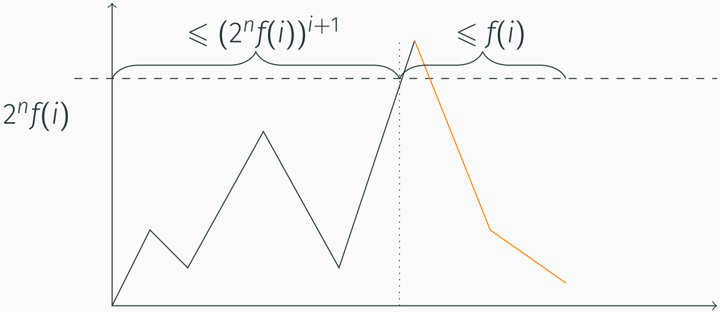

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What do we need to change?

Definition of $f(i)$:
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1st case: Deleting loops creates a subword ✓

2nd case: Replacing second part of the computation ✗

↳ handle with care

Computing the Upward Closure - Upper Bound

Finally:

The minimal words of $\mathcal{L}(N, M_0, M_f)\uparrow$ have a computation of length $\leqslant f(\ell) \leqslant 2^{2^{\mathcal{O}(n \cdot \log n)}}$.

Computing the Upward Closure - Upper Bound

Finally:
The minimal words of $\mathcal{L}(N, M_0, M_f)\uparrow$ have a computation of length $\leqslant f(\ell) \leqslant 2^{2^{\mathcal{O}(n \cdot \log n)}}$.

FSA can simulate the net for $f(\ell)$ steps to accept them (and their upward-closure)

Computing the Upward Closure - Lower Bound

Lemma (Lower Bound)

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Using this idea, we construct for each $n \in \mathbb{N}$ a Petri net with

$$\mathcal{L}(N(n), M_0, M_f) = \left\{ a^{2^{2^n}} \right\} .$$

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We obtain

$$\mathcal{L}(N(n), M_0, M_f)\uparrow = \left\{ a^k \;\middle|\; k \geqslant 2^{2^n} \right\} .$$

Computing the Downward Closure

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Given:	Petri net (N, M_0, M_f).
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Upper bound: *One can compute an FSA of non-primitive recursive size representing the downward closure (in non-primitive recursive time).*

Lower bound: *This is optimal.*

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Proof Sketch.

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The Karp-Miller tree (coverability graph) of the Petri net can be seen as finite automaton *KMT*

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The Karp-Miller tree (coverability graph) of the Petri net can be seen as finite automaton KMT

Its language is a subset of the downward closure,
$\mathcal{L}(N, M_0, M_f) \subseteq \mathcal{L}(KMT) \subseteq \mathcal{L}(N, M_0, M_f)\downarrow$

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$\mathcal{L}(KMT)\downarrow = \mathcal{L}(N, M_0, M_f)\downarrow$

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$\mathcal{L}(KMT)\downarrow = \mathcal{L}(N, M_0, M_f)\downarrow$

Its size might be non-primitive recursive

□

Computing the Downward Closure - Lower Bound

Lemma (Lower Bound)

There is a family of Petri nets such that the downward closure cannot be represented by an FSA of primitive recursive size.

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Inductive construction from [Mayr, Meyer 1981], adapted to labeled Petri nets:

$\forall n, x \in \mathbb{N} \;\; \exists \, (N(n), M_0^{(x)}, M_f)$ polynomial in $(n + x)$ such that

$$\mathcal{L}\left(N(n), M_0^{(x)}, M_f\right) = \left\{a^k \;\middle|\; k \leqslant Acker(n, x)\right\} = \mathcal{L}\left(N(n), M_0^{(x)}, M_f\right)\downarrow$$

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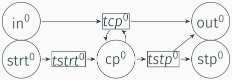

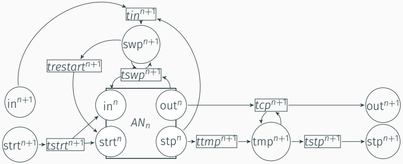

SRE in Downward Closure

Simple Regular Expression

Simple regular expression

$$sre ::= p \mid sre + sre$$
$$p ::= a \mid (a + \varepsilon) \mid \Gamma^* \mid p.p$$

where $\Gamma \subseteq \Sigma$

Known:

Downward and upward closures can be described by SREs

SRE in Downward Closure

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Given: SRE *sre*, Petri net (N, M_0, M_f).

Decide: $\mathcal{L}(sre) \subseteq \mathcal{L}\big(N, M_0, M_f\big)\downarrow$?

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Theorem

SRE in Downward Closure is EXPSPACE-*complete*.

SRE in Downward Closure - Upper Bound

Lemma (Upper Bound)

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Problem: Need to enforce that for each word in Γ^*, a covering computation exists

SRE in Downward Closure - Upper Bound

[Zetzsche 2015]: Downward closures computable iff a certain unboundedness problem decidable

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Theorem (Demri 2013)

The Simultaneous Unboundedness Problem for Petri Nets is EXPSPACE*-complete*.

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Theorem (Demri 2013)

The Simultaneous Unboundedness Problem for Petri Nets is EXPSPACE-complete.

<u>Simultaneous Unboundedness Problem for Petri Nets</u>

Given: Petri net N, marking M_0, set of places $X \subseteq P$

Decide: $\forall n \in \mathbb{N}\ \exists M_0[\sigma\rangle M$ with $M(p) \geqslant n\ \forall p \in X$?

SRE in Downward Closure - Upper Bound

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Handle each product p separately

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For each expression Γ^* in p, add a place that tracks occurrence of all symbols in Γ
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(also track the rest of p)

Check whether the places for the Γ^* are simultaneously unbounded

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Lemma (Lower Bound)

SRE in Downward Closure is EXPSPACE-hard.

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$\mathcal{L}(\emptyset^*) = \{\varepsilon\} \subseteq \mathcal{L}(N, M_0, M_f) \downarrow$ iff M_f coverable

□

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Given: SRE sre, Petri net (N, M_0, M_f).

Decide: $\mathcal{L}(sre) \subseteq \mathcal{L}\big(N, M_0, M_f\big)\!\uparrow$?

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Lower bound (EXPSPACE-hardness) as for SRE in Downward Closure

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Check this using a coverability query in a modified net

Being Downward/Upward closed

Being DC/UC

Being Downward/Upward Closed

Given: Petri net (N, M_0, M_f).

Decide: $\mathcal{L}\big(N, M_0, M_f\big) = \mathcal{L}\big(N, M_0, M_f\big)\downarrow / \uparrow$?

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Being DC and Being UC are decidable.

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$\mathcal{L} \subseteq \mathcal{L}\uparrow$ and $\mathcal{L} \subseteq \mathcal{L}\downarrow$ always hold

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$\mathcal{L}\uparrow$ and $\mathcal{L}\downarrow$ are effectively regular

Regular lang. included in PN coverability lang.

Given: Petri net (N, M_0, M_f), FSA A.

Decide: $\mathcal{L}(A) \subseteq \mathcal{L}\big(N, M_0, M_f\big)$?

REG-IN-PNCOV

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Regular lang. included in PN coverability lang. is decidable.

Theorem (Jancar, Esparza, Moller 1999)

Given Petri net (N, M_0) and FSA A.

$\mathcal{T}(A) \subseteq \mathcal{T}(N, M_0)$ *is decidable.*

Reducing to Trace Inclusion

Theorem (Jancar, Esparza, Moller 1999)

Given Petri net (N', M_0) *and FSA B.*

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$$\mathcal{T}(N', M_0) = \{ w \mid M_0[\sigma\rangle M \text{ for some } M \text{ and } \sigma, \lambda(\sigma) = w \}.$$

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$\mathcal{L}(A) \subseteq \mathcal{L}\big(N, M_0, M_f\big)$ *iff* $\mathcal{T}(A.a) \subseteq \mathcal{T}(N.a, M_0)$.

Reducing to Trace Inclusion

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Lemma

$\mathcal{L}(A) \subseteq \mathcal{L}(N, M_0, M_f)$ *iff* $\mathcal{T}(A.a) \subseteq \mathcal{T}(N.a, M_0)$*.*

where a fresh letter

$A.a$ reduced FSA for $\mathcal{L}(A).a$

$N.a$ = N plus a-labeled transition t_f consuming M_f

BPP Nets

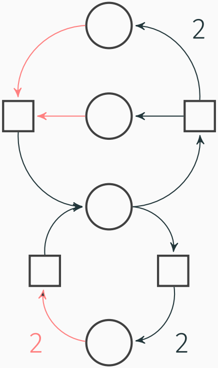
Results

	Petri nets
Compute UC	Doubly exponential*
Compute DC	Non-prim. rec.*
SRE in DC	EXPSPACE-compl.
SRE in UC	EXPSPACE-compl.
Being DC/UC	Decidable

* : Time for construction & size of minimal FSA

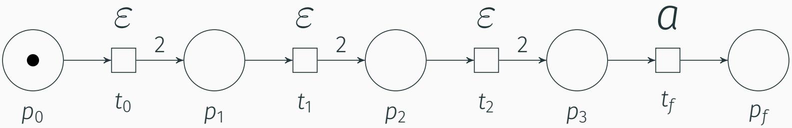
BPP Nets - Negative Example

In a BPP net, each transition consumes at most one token.



BPP Nets - Positive Example

In a BPP net, each transition consumes at most one token.



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Results

	Petri nets	BPP nets
Compute UC	Doubly exponential*	Exponential*
Compute DC	Non-prim. rec.*	Exponential*
SRE in DC	EXPSPACE-compl.	NP-compl.
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Being DC/UC	Decidable	

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Results

	Petri nets	BPP nets	Techniques for upper bound	Techniques for lower bound
Compute UC	Doubly exponential*	Exponential*	Unfoldings	Initial ex.
Compute DC	Non-prim. rec.*	Exponential*	Unfoldings	Initial ex.
SRE in DC	EXPSPACE-compl.	NP-compl.	Presburger	Coverability
SRE in UC	EXPSPACE-compl.	NP-compl.	Coverability	Coverability
Being DC/UC	Decidable			

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Thank you!

Questions?