

On Boundedness in Depth in the π -Calculus

Roland Meyer

University of Oldenburg

2008-02-09

A Client/Server System in the π -Calculus

Client sends on public channel url his private IP address ip to server

Graphically

C — ip

S — l

In π -Calculus

$$\nu ip. \overline{url} \langle ip \rangle . ip(s) . s(x) . C[url, ip] \mid$$
$$\nu l. url(y) . (\nu sn. \bar{y} \langle sn \rangle . T[sn, l] \mid S[url, l])$$

A Client/Server System in the π -Calculus

Client sends on public channel url his private IP address ip to server

Graphically

C — ip

S — I

In π -Calculus

$$\nu ip. \overline{url} \langle ip \rangle . ip(s) . s(x) . C[url, ip] \mid$$
$$\nu l . url(y) . (\nu sn . \bar{y} \langle sn \rangle . T[sn, l] \mid S[url, l])$$

A Client/Server System in the π -Calculus

Client sends on public channel url his private IP address ip to
server

Graphically

C — \boxed{ip}

S — \boxed{I}

In π -Calculus

$\nu ip.\overline{url}\langle ip \rangle.ip(s).s(x).C[url, ip] \mid$

$\nu l.url(y).(\nu sn.\overline{y}\langle sn \rangle.T[sn, I] \mid S[url, I])$

A Client/Server System in the π -Calculus

Client sends on public channel url his private IP address ip to server

Graphically

C — ip

S — l

In π -Calculus

$$\nu ip. \overline{url} \langle ip \rangle . ip(s) . s(x) . C[url, ip] \mid$$
$$\nu l. url(y) . (\nu sn. \bar{y} \langle sn \rangle . T[sn, l] \mid S[url, l])$$

A Client/Server System in the π -Calculus

Client sends on public channel url his private IP address ip to server

Graphically

C — ip

S — l

In π -Calculus

$$\nu ip . \overline{url} \langle ip \rangle . ip(s) . s(x) . C[url, ip] \mid$$
$$\nu l . url(y) . (\nu sn . \bar{y} \langle sn \rangle . T[sn, l] \mid S[url, l])$$

A Client/Server System in the π -Calculus

Server spawns a new thread that handles the session with the client

Graphically

C — ip

S — 1

In π -Calculus

```

$$\nu ip.\overline{url}\langle ip \rangle.ip(s).s(x).C[url, ip] \mid$$

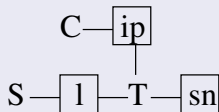
$$\nu l.url(y).( \nu sn.\overline{y}\langle sn \rangle.T[sn, l] \mid S[url, l] )$$

```

A Client/Server System in the π -Calculus

Thread sends back a private session sn on the private channel ip

Graphically



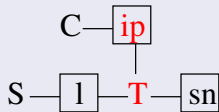
In π -Calculus

$$\nu ip.(ip(s).s(x).C[url, ip] \mid \\ \nu l.(\nu sn.\bar{ip}\langle sn\rangle.T[sn, l] \mid S[url, l]))$$

A Client/Server System in the π -Calculus

Thread sends back a private session sn on the private channel ip

Graphically



In π -Calculus

```

$$\nu ip . ( ip (s) . s(x) . C[url, ip] \mid$$

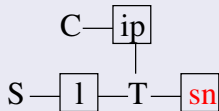
$$\nu l . ( \nu sn . \bar{ip} \langle sn \rangle . T[sn, l] \mid S[url, l] ) )$$

```

A Client/Server System in the π -Calculus

Thread sends back a private session sn on the private channel ip

Graphically



In π -Calculus

```

$$\nu ip.(ip(s).s(x).C[url, ip] \mid$$
  

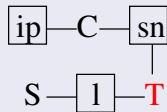
$$\nu l.(\nu sn.\bar{ip}\langle \nu sn \rangle.T[sn, l] \mid S[url, l]))$$

```

A Client/Server System in the π -Calculus

The thread switches its mode

Graphically



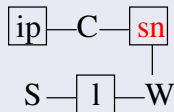
In π -Calculus

$$\nu sn. (\nu ip. sn(x). C[url, ip] \mid \\ \nu l. (T[sn, l] \mid S[url, l]))$$

A Client/Server System in the π -Calculus

To terminate the session, the thread sends the private session object `sn on` the channel `sn` itself

Graphically



In π -Calculus

$$\nu sn . (\nu ip . sn(x) . C[url, ip] \mid \nu l . (\overline{sn} \langle sn \rangle . W[l, sn] \mid S[url, l]))$$

A Client/Server System in the π -Calculus

The thread writes back information to the server and terminates, the client is ready to contact the server again

Graphically

$\boxed{\text{ip}} - \text{C}$

$\text{S} - \boxed{\text{l}} - \text{W} - \boxed{\text{sn}}$

In π -Calculus

$\nu ip. C[url, ip] \mid$

$\nu l. (\nu sn. \text{W}[l, sn] \mid S[url, l])$

A Client/Server System in the π -Calculus

This yields the initial state

Graphically

C — ip

S — l

In π -Calculus

$$\nu ip.C[url, ip] \mid$$
$$\nu l.S[url, l]$$

Motivation and Contribution

Motivation: Verification of dynamically reconfigurable systems

- Does the client/server system terminate?
- Is it finite state?

Contribution: This can be done automatically

- For systems of bounded depth

Overview

- 1 A client/server system in the π -Calculus ✓
- 2 Systems of bounded depth
- 3 From bounded depth to well-structured transition systems
- 4 Decidability results

A normal form for processes

- Normalise the process
 - Minimise the scopes of restrictions
 - Yields parallel composition of fragments F, G

Example

$$\nu l. \nu sn. \nu sn'. (S[url, l] \mid W[l, sn] \mid W[l, sn'])$$

A normal form for processes

- Normalise the process
 - Minimise the scopes of restrictions
 - Yields parallel composition of fragments F, G

Example

$$\nu l. \nu sn. \nu sn'. (S[url, l] \mid W[l, sn] \mid W[l, sn'])$$

A normal form for processes

- Normalise the process
 - Minimise the scopes of restrictions
 - Yields parallel composition of fragments F, G

Example

$$\begin{aligned}
 & \nu l. \nu sn. \nu sn'. (S[url, l] \mid W[l, sn] \mid W[l, sn']) \\
 \equiv & \nu l. \nu sn. (S[url, l] \mid W[l, sn] \mid \nu sn'. W[l, sn'])
 \end{aligned}$$

A normal form for processes

- Normalise the process
 - Minimise the scopes of restrictions
 - Yields parallel composition of fragments F, G

Example

$$\begin{aligned}
 & \nu l. \nu sn. \nu sn'. (S[url, l] \mid W[l, sn] \mid W[l, sn']) \\
 \equiv & \nu l. \nu sn. (S[url, l] \mid W[l, sn] \mid \nu sn'. W[l, sn'])
 \end{aligned}$$

A normal form for processes

- Normalise the process
 - Minimise the scopes of restrictions
 - Yields parallel composition of fragments F, G

Example

$$\begin{aligned} & \nu l. \nu sn. \nu sn'. (S[url, l] \mid W[l, sn] \mid W[l, sn']) \\ \equiv & \nu l. \nu sn. (S[url, l] \mid W[l, sn] \mid \nu sn'. W[l, sn']) \\ \equiv & \nu l. (S[url, l] \mid \nu sn. W[l, sn] \mid \nu sn'. W[l, sn']) \end{aligned}$$

A normal form for processes

- Normalise the process
 - Minimise the scopes of restrictions
 - Yields parallel composition of fragments F, G

Example

$$\begin{aligned}
 & \nu l. \nu sn. \nu sn'. (S[url, l] \mid W[l, sn] \mid W[l, sn']) \\
 \equiv & \nu l. \nu sn. (S[url, l] \mid W[l, sn] \mid \nu sn'. W[l, sn']) \\
 \equiv & \nu l. (S[url, l] \mid \nu sn. W[l, sn] \mid \nu sn'. W[l, sn'])
 \end{aligned}$$

The latter process is a **fragment**

The nesting of restrictions

- Count the nesting of restrictions in the fragment

Example

$$\text{nest}_\nu \left(\nu l. (S[url, l] \mid \nu sn. W[l, sn] \mid \nu sn'. W[l, sn']) \right)$$

The nesting of restrictions

- Count the nesting of restrictions in the fragment

Example

$$\text{nest}_\nu \left(\nu l. (S[url, l] \mid \nu sn. W[l, sn] \mid \nu sn'. W[l, sn'])) \right)$$

$$= 1 + \max\{\dots\}$$

The nesting of restrictions

- Count the nesting of restrictions in the fragment

Example

$$\begin{aligned} & \text{nest}_\nu \left(\nu l. (S[ur\ell, l] \mid \nu sn. W[l, sn] \mid \nu sn'. W[l, sn']) \right) \\ &= 1 + \max\{0, \dots\} \end{aligned}$$

The nesting of restrictions

- Count the nesting of restrictions in the fragment

Example

$$\begin{aligned} & \text{nest}_\nu \left(\nu l.(S[url, l] \mid \nu sn.W[l, sn] \mid \nu sn'.W[l, sn']) \right) \\ &= 1 + \max\{0, 1, \dots\} \end{aligned}$$

The nesting of restrictions

- Count the nesting of restrictions in the fragment

Example

$$\begin{aligned}
 & \text{nest}_\nu \left(\nu l. (S[url, l] \mid \nu sn. W[l, sn] \mid \nu sn'. W[l, sn']) \right) \\
 &= 1 + \max\{0, 1, 1\} \\
 &= 2
 \end{aligned}$$

The nesting of restrictions

- Count the nesting of restrictions in the fragment
- Problem: The fragment representation is not unique

Example

$$\begin{aligned} & \nu l.(S[url, l] \mid \nu sn.W[l, sn] \mid \nu sn'.W[l, sn']) =: F \\ \equiv & \nu sn.\nu sn'.\nu l.(S[url, l] \mid W[l, sn] \mid W[l, sn']) =: G \end{aligned}$$

Then $nest_{\nu}(F) = 2$ but $nest_{\nu}(G) = 3$

The nesting of restrictions

- Count the nesting of restrictions in the fragment
- Problem: The fragment representation is not unique

Example

$$\begin{aligned} & \nu l.(S[url, l] \mid \nu sn.W[l, sn] \mid \nu sn'.W[l, sn']) =: F \\ \equiv & \nu sn.\nu sn'.\nu l.(S[url, l] \mid W[l, sn] \mid W[l, sn']) =: G \end{aligned}$$

Then $nest_{\nu}(F) = 2$ but $nest_{\nu}(G) = 3$

Boundedness in depth

- Solution: Define the **depth** of a fragment as the nesting of restrictions in the **flattest representation**

Depth

$$\text{depth}(F) = \min \{ \text{nest}_\nu(G) \mid G \equiv F \}$$

Boundedness in depth

- Solution: Define the depth of a fragment as the nesting of restrictions in the flattest representation

Depth

$$\text{depth}(F) = \min\{\text{nest}_\nu(G) \mid G \equiv F\}$$

- A process is **bounded in depth** if the depth of **all reachable fragments** is bounded

A Characterisation of Boundedness in Depth

- Problem: No good intuition to processes of bounded depth
- How to find the flat representation?

A Characterisation of Boundedness in Depth

Theorem

A process is bounded in depth if and only if

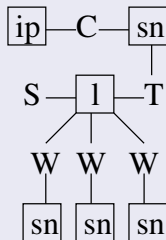
the length of the longest simple paths in the graphs is bounded

- Simple paths do not repeat hyperedges
- Reachable states are star-like
- Anchored fragments are flat representations

Example: The Client/Server System

- In the case study, the depth is bounded by 4

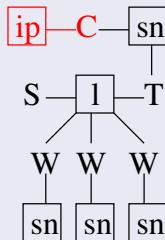
A longest simple path



Example: The Client/Server System

- In the case study, the depth is bounded by 4

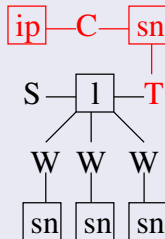
A longest simple path



Example: The Client/Server System

- In the case study, the depth is bounded by 4

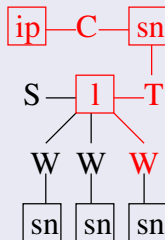
A longest simple path



Example: The Client/Server System

- In the case study, the depth is bounded by 4

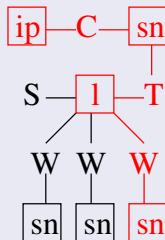
A longest simple path



Example: The Client/Server System

- In the case study, the depth is bounded by 4

A longest simple path



More Examples

All decidable subclasses of π -Calculus are bounded in depth

- Finitary agents [FGMP03], finite control processes [Dam96], bounded processes [Cai04] (finite state systems)
- Structurally stationary processes [Mey08], finite handler processes [Mey08], restriction-free processes [AM02] (Petri nets)
- Bounded input unique receiver systems [AM02] (subclass of transfer nets)
- Finite net processes [BG95, BG08] (subclass of inhibitor nets)

Well-Structured Transition Systems

- Framework for infinite state systems [Fin90, FS01, AČJT00]
- Generalises decidability results for particular models

Technically

WSTS= (S, \rightarrow, \leq) where

- (S, \rightarrow) is a transition system
- $\leq \subseteq S \times S$ is an ordering on the states with two properties

Well-Structured Transition Systems

- Framework for infinite state systems [Fin90, FS01, AČJT00]
- Generalises decidability results for particular models

Technically

WSTS = (S, \rightarrow, \leq) where

- (S, \rightarrow) is a transition system
- $\leq \subseteq S \times S$ is an ordering on the states with two properties

Well-Structured Transition Systems

- Framework for infinite state systems [Fin90, FS01, AČJT00]
- Generalises decidability results for particular models

Technically

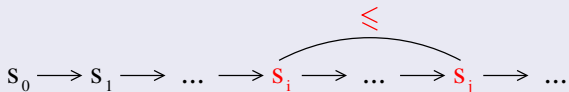
WSTS = (S, \rightarrow, \leq) where

- (S, \rightarrow) is a transition system
- $\leq \subseteq S \times S$ is an ordering on the states with two properties

Well-Structured Transition Systems

$\leq \subseteq S \times S$ is a well-quasi-ordering

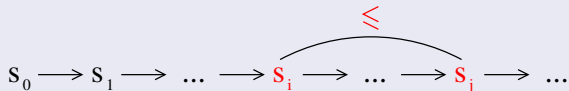
In every infinite sequence of states, there are two comparable ones



Well-Structured Transition Systems

$\leq \subseteq S \times S$ is a well-quasi-ordering

In every infinite sequence of states, there are two comparable ones



$\leq \subseteq S \times S$ is a **simulation**

Larger states can imitate the transition behaviour of smaller ones



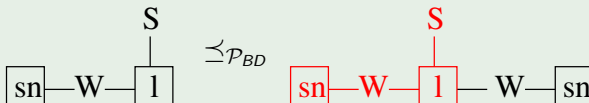
Instantiation of the framework—the ordering $\preceq_{\mathcal{P}_{BD}}$

- Intuitively: Hypergraph embedding so that
 - No connections are added to vertices
 - No connections are removed from vertices

Instantiation of the framework—the ordering $\preceq_{\mathcal{P}_{BD}}$

- Intuitively: Hypergraph embedding so that
 - No connections are added to vertices
 - No connections are removed from vertices

Example



Instantiation of the framework—the ordering $\preceq_{\mathcal{P}_{BD}}$

- Intuitively: Hypergraph embedding so that
 - No connections are added to vertices
 - No connections are removed from vertices
- Technically: **Parallel composition** of fragments **may be added**

$$\nu a.(F \mid G) \preceq_{\mathcal{P}_{BD}} \nu a.(F \mid G \mid H)$$

Instantiation of the framework—the ordering $\preceq_{\mathcal{P}_{BD}}$

- Intuitively: Hypergraph embedding so that
 - No connections are added to vertices
 - No connections are removed from vertices
- Technically: Parallel compositions of fragments may be added

$$\nu a.(F \mid G) \preceq_{\mathcal{P}_{BD}} \nu a.(F \mid G \mid H)$$

Example

$$\begin{aligned} & \nu l.(S[url, l] \mid \nu sn.W[l, sn]) \\ \preceq_{\mathcal{P}_{BD}} & \nu l.(S[url, l] \mid \nu sn.W[l, sn] \mid \nu sn'.W[l, sn']) \end{aligned}$$

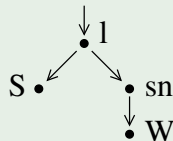
Why is it a wqo?

- Understand fragments as (syntax) trees
 - Processes are leafs
 - Restricted names are nodes

Example

$\nu l.(S[url, l] \mid \nu sn.W[l, sn])$

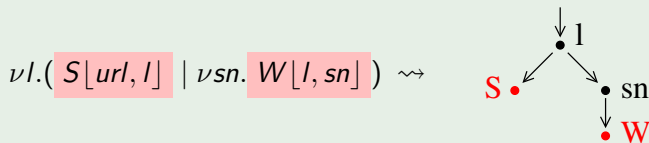
\rightsquigarrow



Why is it a wqo?

- Understand fragments as (syntax) trees
 - Processes are **leafs**
 - Restricted names are nodes

Example

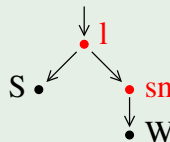


Why is it a wqo?

- Understand fragments as (syntax) trees
 - Processes are leafs
 - Restricted names are **nodes**

Example

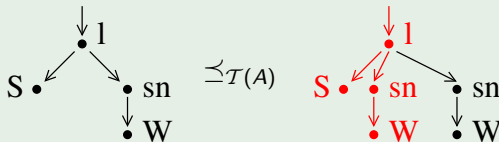
$\nu l . (S[url, l] \mid \nu sn . W[l, sn]) \rightsquigarrow$



Why is it a wqo?

- Understand fragments as (syntax) trees
 - Processes are leafs
 - Restricted names are nodes
- Use a suitable wqo on trees

Example



- Wqo on trees of bounded depth
- Induction on depth + Higman's result [Hig52]

Main Result

- Simulation is easier to prove

Theorem

If P is a process of bounded depth, then $(\text{Reach}(P)/\equiv, \rightarrow, \preceq_{\mathcal{P}_{BD}})$ is a well-structured transition system.

Decidability Results for WSTS [Fin90, FS01, AČJT00]

- Build the computation tree (finite branching)
- If a new node covers a predecessor stop the computation, mark the node by +

Decidability Results for WSTS [Fin90, FS01, AČJT00]

- Build the computation tree (finite branching)
- If a new node covers a predecessor stop the computation, mark the node by +

Theorem ([Fin90, FS01, AČJT00])

There is a non-terminating computation if and only if the tree contains a node marked by +.

Decidability Results for WSTS [Fin90, FS01, AČJT00]

- Build the computation tree (finite branching)
- If a new node covers a predecessor stop the computation, mark the node by +

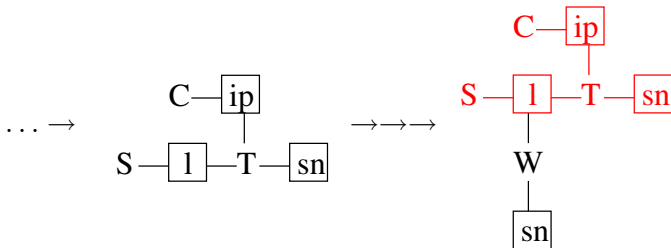
Theorem ([Fin90, FS01, AČJT00])

There is a non-terminating computation if and only if the tree contains a node marked by +.

Infinite state if and only if + node is truly bigger

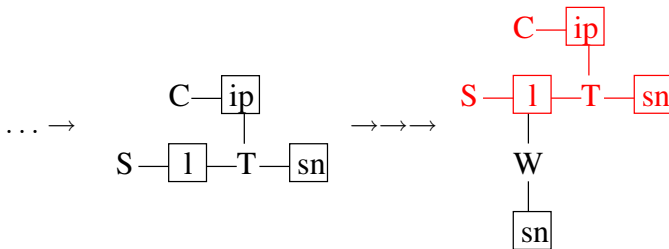
Application to the Client/Server System

- Build the computation tree



Application to the Client/Server System

- Build the computation tree



Results

The system does **not terminate**, the system is **infinite state**

Related Work

- Interpretation of processes as graphs due to Milner (flow graphs) [Mil79, MM79]
 - For π -Calculus in [MPW92, Mil99, SW01]
 - We relate **depth on terms** with the **longest simple paths in graphs**
- Normal forms for π -Calculus by Engelfriet and Gelsema [EG99, EG04] and Milner [Mil99]
 - Similar to minimising scopes
 - **Anchored fragments** are more stringent

Related Work

- WSTS by Finkel [Fin90, FS01] and Abdulla et. al. [AČJT00]
 - Finkel inspired by Petri nets, termination and boundedness problems
 - Abdulla inspired by lossy channel systems, temporal and simulation properties
 - First **instantiation** for π -Calculus
 - $\preceq_{\mathcal{P}_{BD}}$ is non-trivial
- Importance of termination for π -Calculus by Yoshida et. al. [YBH04] and Sangiorgi [DS06]
 - Type systems that ensure termination of well-typed processes
 - Instantiate WSTS framework, derive **decidability** of termination as **corollary**

Conclusion and Thanks

Processes of bounded depth are graphs where the longest simple path is bounded

- Star-like structures
- Unbounded parallelism/unboundedly many restricted names

They have well-structured transition systems

- Termination is decidable
- Infinity of states is decidable

The class is huge

- Contains all decidable subclasses of π -Calculus known so far [BG95, Dam96, AM02, FGMP03, Cai04, Mey08, BG08]

Conclusion and Thanks

Thanks for your attention

References I



P. A. Abdulla, K. Čerans, B. Jonsson, and Y.-K. Tsay.

Algorithmic analysis of programs with well quasi-ordered domains.
Information and Computation, 160(1–2):109–127, 2000.



R. M. Amadio and C. Meyssonnier.

On decidability of the control reachability problem in the asynchronous π -calculus.
Nordic Journal of Computing, 9(1):70–101, 2002.



N. Busi and R. Gorrieri.

A Petri net semantics for π -calculus.

In *Proc. of the 6th International Conference on Concurrency Theory, CONCUR 1995*, volume 962 of LNCS, pages 145–159. Springer-Verlag, 1995.



N. Busi and R. Gorrieri.

Distributed semantics for the π -calculus based on Petri nets with inhibitor arcs.
To appear in JLAP, August 2008.



L. Caires.

Behavioural and spatial observations in a logic for the π -Calculus.

In *Proc. of the 7th International Conference on Foundations of Software Science and Computation Structures, FOSSACS 2004*, volume 2987 of LNCS, pages 72–89. Springer-Verlag, 2004.

SPATIAL LOGIC MODEL CHECKER: <http://ctp.di.fct.unl.pt/SLMC/>.

References II



M. Dam.

Model checking mobile processes.

Information and Computation, 129(1):35–51, 1996.



Y. Deng and D. Sangiorgi.

Ensuring termination by typability.

Information and Computation, 204(7):1045–1082, 2006.



J. Engelfriet and T. Gelsema.

Multisets and structural congruence of the pi-calculus with replication.

Theoretical Computer Science, 211(1-2):311–337, 1999.



J. Engelfriet and T. Gelsema.

A new natural structural congruence in the pi-calculus with replication.

Acta Informatica, 40(6):385–430, 2004.



G.-L. Ferrari, S. Gnesi, U. Montanari, and M. Pistore.

A model-checking verification environment for mobile processes.

ACM Transactions on Software Engineering and Methodology, 12(4):440–473, 2003.

HAL: <http://fmt.isti.cnr.it:8080/hal/>.



A. Finkel.

Reduction and covering of infinite reachability trees.

Information and Computation, 89(2):144–179, 1990.

References III



A. Finkel and Ph. Schnoebelen.
Well-structured transition systems everywhere!
Theoretical Computer Science, 256(1–2):63–92, 2001.



G. Higman.
Ordering by divisibility in abstract algebras.
Proc. London Math. Soc. (3), 2(7):326–336, 1952.



R. Meyer.
A theory of structural stationarity in the π -calculus.
55 pages, submitted for publication, June 2008.



R. Milner.
Flowgraphs and flow algebras.
Journal of the Association for Computing Machinery, 26(4):794–818, 1979.



R. Milner.
Communicating and Mobile Systems: the π -Calculus.
Cambridge University Press, 1999.



G. Milne and R. Milner.
Concurrent processes and their syntax.
Journal of the Association for Computing Machinery, 26(2):302–321, 1979.

References IV



R. Milner, J. Parrow, and D. Walker.

A calculus of mobile processes, part I.

Information and Computation, 100(1):1–40, 1992.



D. Sangiorgi and D. Walker.

The π -calculus: a Theory of Mobile Processes.

Cambridge University Press, 2001.



N. Yoshida, M. Berger, and K. Honda.

Strong normalisation in the π -Calculus.

Information and Computation, 191(2):145–202, 2004.