The Downward-Closure of Petri Net Languages

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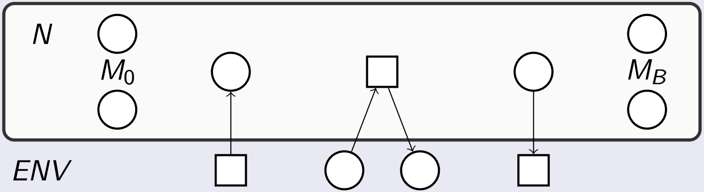
June 2010

Stability Analysis

Problem



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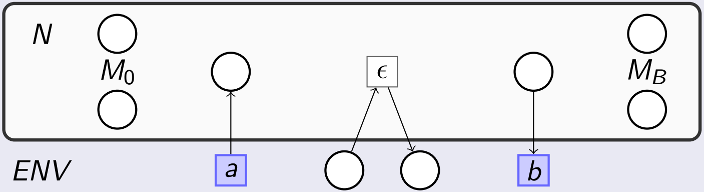

Goal

Determine influences N can tolerate without reaching M_B from M_0

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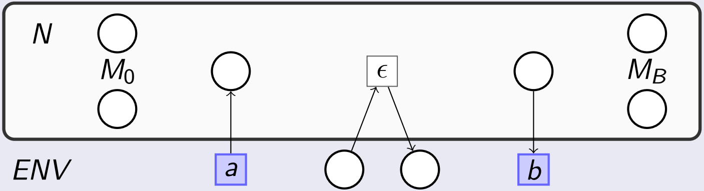

Approximate environmental behaviour $\mathcal{L}_h(N, M_0, M_B)$

Scattered embedding [Hig52] forgets letters

$$\mathcal{L}_h(N, M_0, M_B) \subseteq \mathcal{L}_h(N, M_0, M_B)\downarrow$$

Stability Analysis

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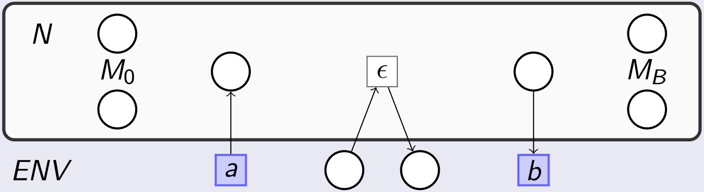

Approximate environmental behaviour $\mathcal{L}_h(N, M_0, M_B)$

$\mathcal{L}\downarrow$ regular:

Stability Analysis

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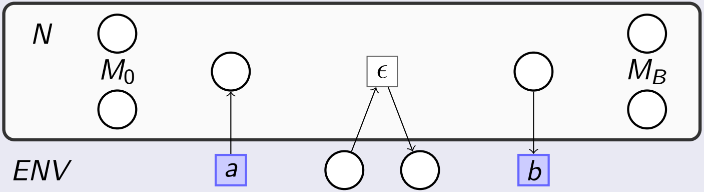

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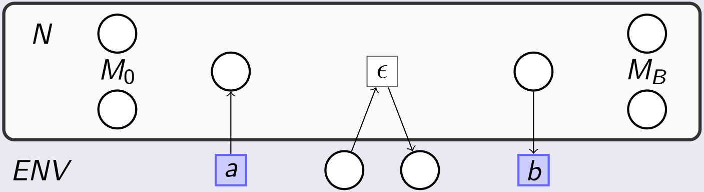

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$\mathcal{L}\downarrow$ regular: complement upward-closed, finite basis by wqo

$$\overline{\mathcal{L}_h(N, M_0, M_B)\downarrow} \subseteq \overline{\mathcal{L}_h(N, M_0, M_B)}$$

Contribution

Result

Downward-closure of Petri net languages computable

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Language types

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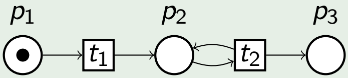
Further applications

- Analysis of asynchronous systems
- Compositional verification
- Regular approximation

Petri Nets and Coverability Graphs

Petri net

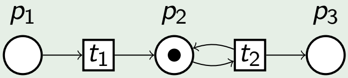


Its coverability graph [KM69]

$$\downarrow$$
$$(1, 0, 0)$$

Petri Nets and Coverability Graphs

Petri net



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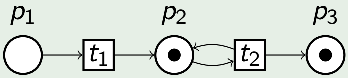
$$(1, \overset{\downarrow}{0}, 0) \xrightarrow{t_1} (0, 1, 0)$$

Petri Nets and Coverability Graphs

Petri net



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$$(1,0,0) \xrightarrow{t_1} (0,1,0) \xrightarrow{t_2} (0,1,1)$$

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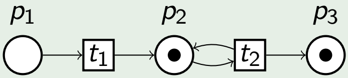

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$$(1,0,0) \xrightarrow{t_1} (0,1,0) \xrightarrow{t_2} (0,1,\omega)$$

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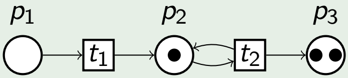

Its coverability graph [KM69]

$$(1,0,0) \xrightarrow{t_1} (0,1,0) \xrightarrow{t_2} (0,1,\omega) \circlearrowleft^{t_2}$$

Main Result

Definition (Ordinary Petri net language in *PN*)

$$\mathcal{L}_h(N, M_0, M_f) := \{h(\sigma) \mid M_0[\sigma\rangle M_f\}$$

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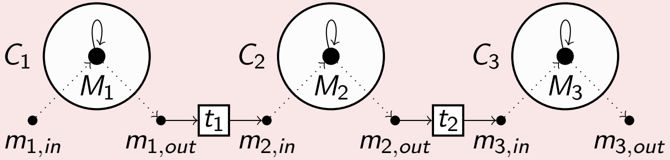
Theorem (Representation)

A regular expression ϕ is computable with

$$\mathcal{L}_h(N, M_0, M_f)\downarrow = \phi$$

Lambert's Marked Graph Transition Sequences [Lam92]

Marked graph transition sequence $G = C_1.t_1.C_2.t_2.C_3$



Properties

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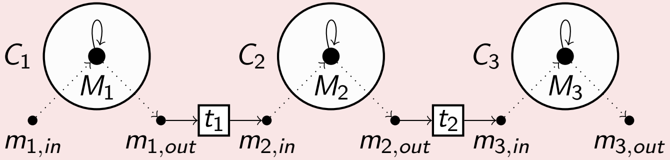

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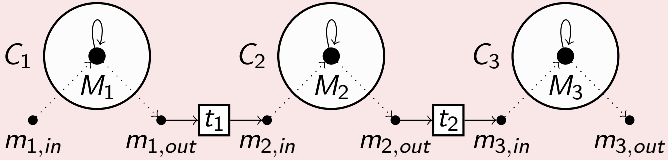

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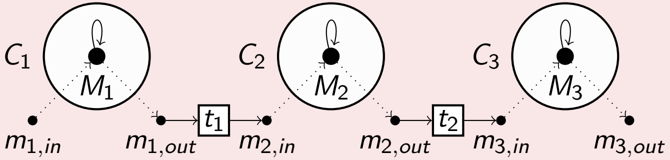

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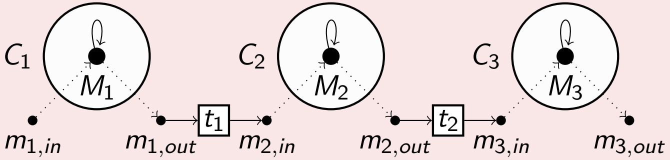

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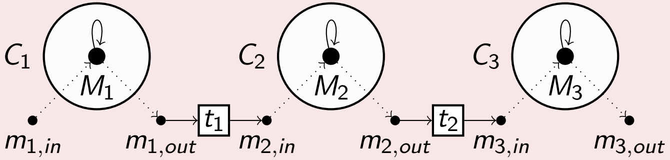

Properties

- C strongly connected coverability graph
- M initial marking, m_{in} input marking , m_{out} output marking
- Input and output less abstract $\preceq_\omega$ than initial marking

$$m_{in} \preceq_\omega M \quad \text{if} \quad m_{in}(p) = M(p) \quad \text{or} \quad M(p) = \omega$$

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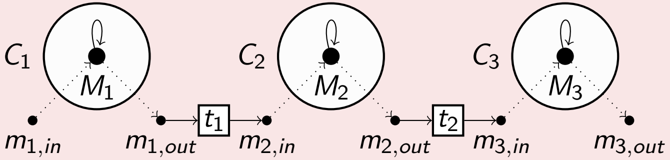

Solutions $\mathcal{L}(G)$

Transition sequence σ through $G = C_1.t_1.C_2.t_2.C_3$

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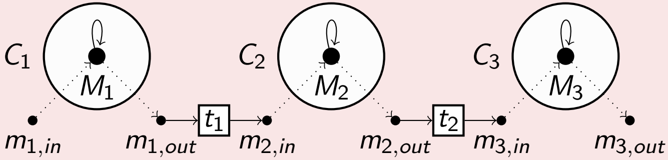

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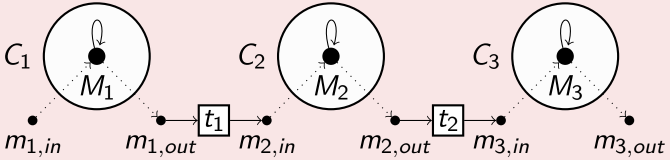

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- Start in $m_{1,in}$, fire transitions in C_1

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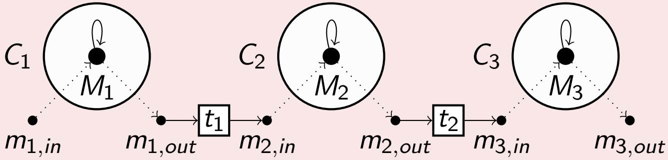

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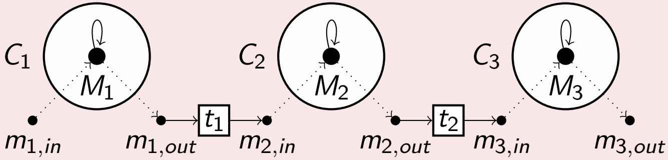

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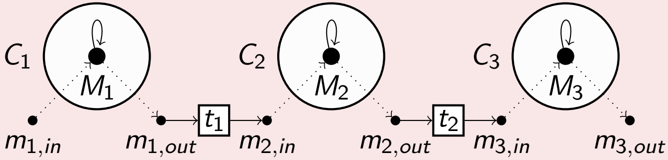

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Concrete values must be reached exactly!

MGTS and Reachability

Reachability problem $RP = (N, M_0, M_f)$

Is M_f reachable from M_0 in N?

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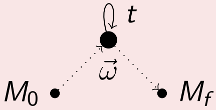
Corresponding mgts G_{RP}



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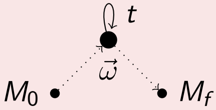

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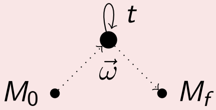

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Even more

$$\mathcal{L}(N, M_0, M_f) = \{\sigma \mid M_0[\sigma\rangle M_f\} = \mathcal{L}(G_{RP})$$

Characteristic Equation

Characteristic equation for mgts

Encode firing in $G = C_1.t_1.C_2.t_2.C_3$ into $Ax = b$

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They are computable! [Lam88]

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Relationship

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$$\Leftarrow \quad \text{does not hold \ldots in general}$$

Perfect MGTS

Perfect mgts $\mathbb{G} = C_1.t_1.C_2.t_2.C_3$

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But not every mgts is perfect!

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Idea

Improve perfectness by unrolling

Perfect MGTS

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Property

$\mathcal{L}(\mathbb{G}) \neq \emptyset \quad \Leftrightarrow \quad Ax = b$ solvable (later)

Theorem (Lambert's decomposition theorem [Lam92])

G can be effectively decomposed into a finite set Γ_G of perfect mgts with

$$\mathcal{L}(G) = \bigcup_{\mathbb{G} \in \Gamma_G} \mathcal{L}(\mathbb{G})$$

Solving Reachability

Where are we?

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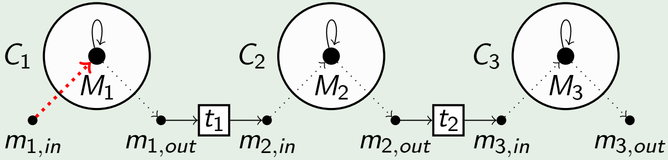
Goal

Compute $\sigma \in \mathcal{L}(\mathbb{G})$ from solution to $Ax = b$

Solving Reachability

Perfect mgts



Pumping tokens

Sequence u leads from m_{in} to M

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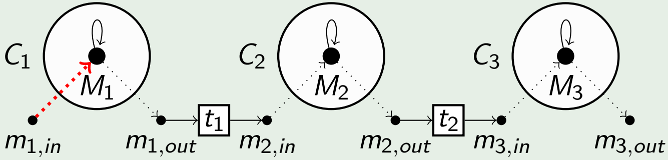

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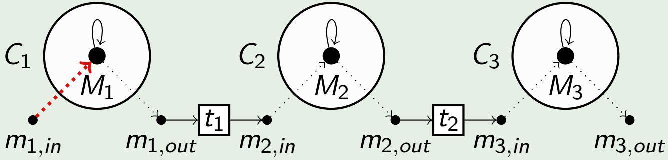

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- u does not change tokens on p with $M(p) = m_{in}(p) = k \in \mathbb{N}$

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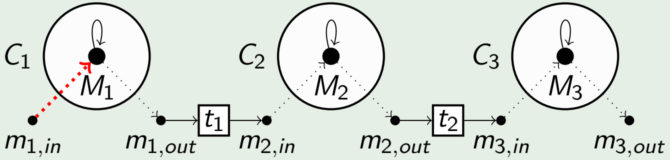

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u is computable! [Lam92]

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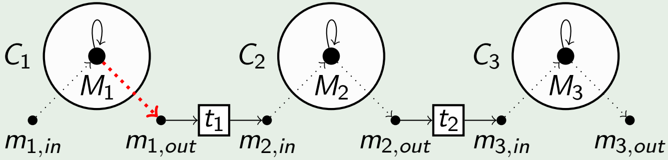


Pumping tokens

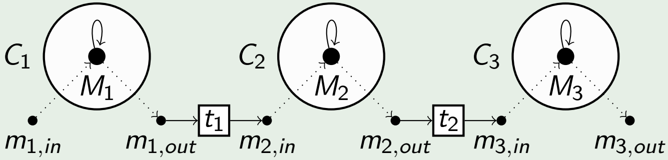
Sequence u leads from m_{in} to M

- u adds tokens to p with $M(p) = \omega > m_{in}(p)$
- u does not change tokens on p with $M(p) = m_{in}(p) = k \in \mathbb{N}$

Sequence v leads from M to m_{out}

Solving Reachability

Perfect mgts



Theorem (Lambert's pumping lemma)

Let $Ax = b$ have solution. Then $\sigma_k \in \mathcal{L}(\mathbb{G})$ for every $k > k_0$ with

$$\sigma_k = (u_1^k.\beta_1.\alpha_1^k.v_1^k).t_1.(u_2^k.\beta_2.\alpha_2^k.v_2^k)\ldots t_{n-1}.(u_n^k.\beta_n.\alpha_n^k.v_n^k)$$

Computing the Downward-Closure

Computing the downward-closure

$\mathcal{L}(N, M_0, M_f)$

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Let $\mathbb{G} = C_1.t_1.C_2 \ldots t_{n-1}.C_n$ *have solution. Then*

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$$T^* \subseteq \bigcup_{k\geq k_0} u^k\downarrow$$

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$$\begin{aligned} & T_1^*.(t_1+\epsilon).T_2^* \ldots (t_{n-1}+\epsilon).T_n^* \\ \subseteq\ & \bigcup_{k\geq k_0} (u_1^k.\beta_1.\alpha_1^k.v_1^k).t_1.(u_2^k.\beta_2.\alpha_2^k.v_2^k) \ldots t_{n-1}.(u_n^k.\beta_n.\alpha_n^k.v_n^k)\downarrow \end{aligned}$$

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- Take u_b with all transitions T in C (strongly connected)

$$u := u_a^{m+1}.u_b \qquad m = \text{maximal negative effect of } u_b$$

Terminal Languages

Definition (Terminal language in *TPN*)

$$\mathcal{T}_h(N, M_0) = \{h(\sigma) \mid M_0[\sigma\rangle M \text{ and } \neg M[t\rangle \text{ f.a. } t \in T\}$$

Terminal Languages

Idea

Deadlocks given by finite set $\mathcal{P}$ of partial markings M_P, $P \subseteq P'$

$$\mathcal{T}_h(N, M_0) = \bigcup_{M_P \in \mathcal{P}} \mathcal{L}_h(N, M_0, M_P)$$

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Theorem (Representation)

$$\mathcal{T}_h(N, M_0)\downarrow = \Sigma_{M_P \in \mathcal{P}} \quad \phi_{M_P}$$

Partial Languages and their Representation

Partial languages: acceptance by corresponding markings

$$\mathcal{L}_h(N, M_0, M_P) = \bigcup_{M|_P = M_P} \mathcal{L}_h(N, M_0, M)$$

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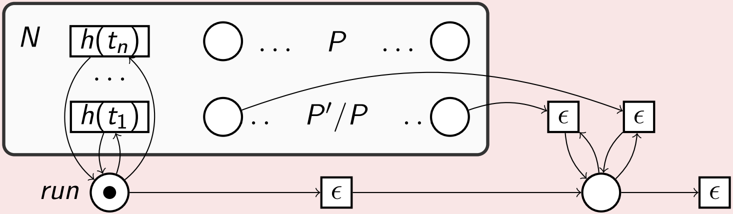
From infinite union to ordinary language [Hac76]



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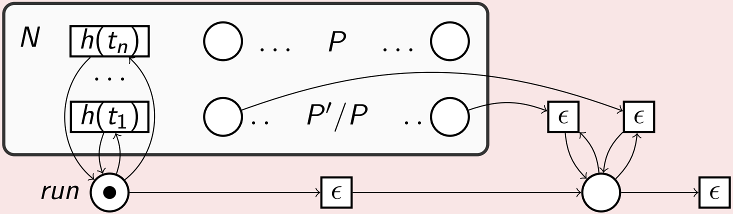


Lemma

$$\mathcal{L}_h(N, M_0, M_P) = \mathcal{L}_{h \cup h_g}(N, M_0^{run}, M_P^{empty})$$

Covering Languages

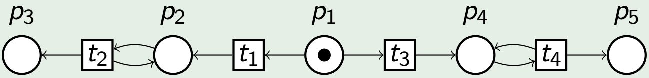
Definition (Covering language in *CPN*)

$$\mathcal{C}_h(N, M_0, M_f) = \{h(\sigma) \mid M_0[\sigma\rangle M \text{ and } M \geq M_f\}$$

Covering Languages

Turn coverability tree into finite automaton

Example



$$\begin{array}{ccccc} (0,1,0,0,0) & \xleftarrow{t_1} & (1,0,0,0,0) & \xrightarrow{t_3} & (0,0,0,1,0) \\ t_2 \downarrow & & & & \downarrow t_4 \\ (0,1,\omega,0,0) & & & & (0,0,0,1,\omega) \end{array}$$

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- Silent transitions to smaller nodes

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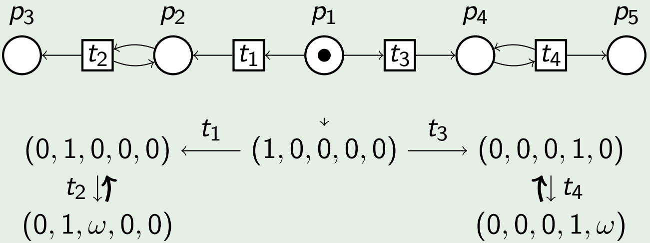

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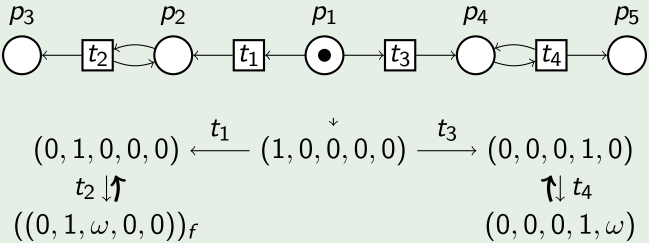

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Lemma

$$\mathcal{C}_h(N, M_0, M_f)\downarrow = \mathcal{L}(FA)\downarrow$$

Downward-closure of the Automaton

Tree of strongly connected components

Example

$$
\begin{array}{ccccc}
(0,1,0,0,0) & \xleftarrow{t_1} & (1,0,\overset{\downarrow}{0},0,0) & \xrightarrow{t_3} & (0,0,0,1,0) \\
t_2 \downarrow\!\uparrow & & & & \uparrow\!\downarrow t_4 \\
((0,1,\omega,0,0))_f & & & & (0,0,0,1,\omega)
\end{array}
$$

Downward-closure of the Automaton

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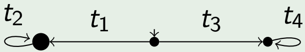

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Compute expression recursively

$$\phi_C = T_C^*.(\qquad\qquad\qquad\qquad)$$

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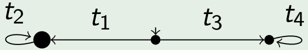

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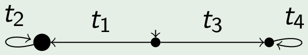

Downward-closure of the Automaton

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$$\phi_C = T_C^*.(\tau_C \quad + \quad \Sigma_{C'} \quad \gamma_{C,C'}.\phi_{C'})$$

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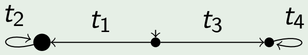

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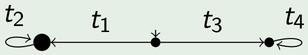

$$\phi_{C_0} = (t_1 + \epsilon).t_2^* \quad + \quad (t_3 + \epsilon).t_4^*.\emptyset$$

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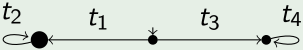

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Van Leeuwen's Algebraic Languages

Definition (Algebraic languages over K)

- *K-grammar* is (V, Σ, P, S) with productions P of form

$$A \rightarrow \mathcal{L} \in K \qquad \mathcal{L} \subseteq V^*$$

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$REG^\nabla = CF$

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Example (Context-free languages)

$REG^\nabla = CF$

Theorem (Van Leeuwen's Theorem 4.5 [vL78])

$\mathcal{L}\downarrow$ effectively computable for all $\mathcal{L} \in K^\nabla$ iff so for all $\mathcal{L} \in K$

Consequences

Consequence of van Leeuwen's work and our results

$\mathcal{L}\downarrow$ effectively computable for $\mathcal{L}$ in PN^{∇}, TPN^{∇}, and CPN^{∇}

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Example (Language in PN^∇)

$$\underbrace{u_1.u_1^R \ldots u_n.u_n^R}_{\sim A^n} . \quad \underbrace{v_1.v_1^R \ldots v_n.v_n^R}_{\sim B^n} . \quad \underbrace{w_1.w_1^R \ldots w_n.w_n^R}_{\sim C^n}$$

Conclusion

Downward-closure of all Petri net languages computable

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- Ordinary languages via mgts and Lambert's pumping lemma for reachability

Conclusion

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Applications

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- Compositional verification

Related Work

- Upward/Downward-closure of context-free languages [vL78]

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- Size of automata representation [GHK07, GHK09]

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