SS 2013 June 5th, 2013

In-class Exercises to the Lecture Logics
Sheet 4

Jun.-Prof. Dr. Roland Meyer Discussion on June 6th/7th, 2013

Exercise 4.1 [Resolution]
Prove by resolution that the following formula is unsatisfiable:

$$(\neg p \vee s) \wedge (p \vee q \vee \neg r) \wedge (r \vee q \vee s) \wedge (q \vee \neg s) \wedge (\neg q \vee s) \wedge (\neg q \vee \neg s)$$

Exercise 4.2 [Formulae in 2CNF]
A formula is in 2CNF if it is of the form $C_1 \wedge \cdots \wedge C_n$, where each C_i is a clause with at most two literals. Devise a method to check satisfiability for such formulae that runs in polynomial time.

Exercise 4.3 [Complete sets of connectives]

a) Prove that for any formula $A \in F(\{\vee\})$, we have $\varphi(A) = 0$, where φ is the valuation with $\varphi(p) = 0$ for each atomic formula p.
b) Show that $\{\vee\}$ is *not* a complete set of connectives.

Exercise 4.4 [Predicate Logic]
Suppose the signature S contains the predicates p_{IsFish} and p_{CanSwim}. Let $A \equiv \forall x(p_{\text{CanSwim}}(x) \rightarrow p_{\text{IsFish}}(x))$.
a) Present a structure $\mathcal{M}$ over the signature S with $\mathcal{M} \models A$.
b) Present a structure $\mathcal{M} = (D, I)$ over the signature S with $\mathcal{M} \models A$, where $D = \{\text{Duck}, \text{Herring}, \text{Carp}\}$.