SS 2013 May 22, 2013

In-class Exercises to the Lecture Logics
Sheet 3

Jun.-Prof. Dr. Roland Meyer Discussion on May 23 and 24, 2013

Exercise 3.1 [König's Lemma I]

Smullyan's ball game is played by one player according to the following rules: There is a container that has capacity for an unlimited number of balls. Furthermore, there is an unlimited supply of balls, each of which is labeled with a natural number. Initially, there is one ball in the container. In each step, the player can remove a ball from the container and put in an arbitrary (finite) number of balls, provided that each of the new balls has a label smaller than the removed ball. The game is over when no further step is possible, i.e. when the container is empty.

Show that each game is over after a finite number of steps.

Exercise 3.2 [König's Lemma II]

Let Σ be a finite set of symbols. A *word over* Σ is a finite sequence of symbols from Σ. An *infinite word over* Σ is an infinite sequence of symbols from Σ. Let L be a prefix-closed set of words over Σ, i.e.: If a word is contained in L, then each of its prefixes is as well. Prove: If L is infinite, then there is an infinite word over Σ each of whose (finite) prefixes is contained in L.

Exercise 3.3 [The subsumption rule for the Davis-Putnam method]

Let $F = \{K_1, \ldots, K_n\}$ be a formula in KNF in set notation. Moreover, let $K_i \subseteq K_j$. Show that $F \models\!\!\!\!= F'$, in which $F' = F \backslash \{K_j\}$.

Exercise 3.4 [Davis Putnam Method]

Using the Davis Putnam Method, determine whether the following formula is satisfiable:

$$\neg p \wedge (\neg t \vee q \vee p) \wedge (\neg s \vee \neg q) \wedge (\neg s \vee \neg q \vee t)$$