Exercises to the lecture Logics
Sheet 7

Jun.-Prof. Dr. Roland Meyer Due July 19, 2013, 12:00pm

Exercise 7.1 [Tableaux for predicate logic]
Using tableaux, determine whether

a) the formula $\forall z \exists x \exists y p(x, y, z)$ is satisfiable.
b) the formula $\forall x \forall y\, ((p(x, y) \wedge q(x)) \rightarrow \exists y q(y))$ is a tautology.

Exercise 7.2 [Decidable theories]
Let T be a recursively decidable theory. Show that there is a recursively enumerable system of axioms that generates T.

Exercise 7.3 [The deductive system $\mathcal{F}$]
Prove:

a) $\forall x[p(x, y)],\ y = z \vdash_{\mathcal{F}} \forall x[p(x, z)]$.

b) $\forall x[p(x) \rightarrow q(x)], \forall x[p(x)] \vdash_{\mathcal{F}} q(f(a))$

Exercise 7.4 [Theory of equality]
Consider a signature S that only contains function symbols. The theory of equality T_{Σ_E} is defined by the following set of axioms Σ_E:

$$\forall x.x = x \qquad \forall x \forall y.x = y \rightarrow y = x \qquad \forall x \forall y \forall z.x = y \wedge y = z \rightarrow x = z$$
$$\forall x_1 \ldots \forall x_n \forall y_1 \ldots \forall y_n. \bigwedge_{i=1}^{n} x_i = y_i \rightarrow f(x_1, \ldots, x_n) = f(y_1, \ldots, y_n),$$

where $f/n \in S$. Develop an algorithm that checks whether a quantifier-free and closed formula A is contained in T_{Σ_E}. As an example, your algorithm should return

$$a = b \wedge f(a) = f(g(b)) \rightarrow f(b) = f(g(a)) \in T_{\Sigma_E}$$
$$a = b \wedge \neg(f(a) = f(b)) \notin T_{\Sigma_E}.$$

Delivery: until July 19, 2013, 12:00pm into the box next to room 34/401.4