SS 2013 15.5.2013

Exercises to the lecture Logics
Sheet 3

Jun.-Prof. Dr. Roland Meyer Due 24.5.2013 12:00 Uhr

Exercise 3.1 [Proofs in the calculus $\mathcal{F}_0$]

Prove:

a) $\neg(p \rightarrow q) \vdash_{\mathcal{F}_0} q \rightarrow p$

b) ~~$r \rightarrow (p \rightarrow \neg p) \vdash_{\mathcal{F}_0} \neg r$~~ $r \rightarrow \neg(p \rightarrow p) \vdash_{\mathcal{F}_0} \neg r$

c) $p \rightarrow (q \rightarrow r) \vdash_{\mathcal{F}_0} \neg r \rightarrow (q \rightarrow \neg p)$

You can use all theorems in the old lecture notes, in-class Exercise 2.1, and the deduction theorem but not the completeness of $\mathcal{F}_0$.

Exercise 3.2 [Complete open branches in tableaux]

Prove Lemma 2.10 on the old slides using structural induction.

Exercise 3.3 [Tableaux consequence]

For a set of formulae Σ and a formula A, we write $\Sigma \vdash_{\tau} A$ if there is a closed tableau for the set $\Sigma \cup \{\neg A\}$. Prove:

a) $A \wedge \neg B \vdash_{\tau} \neg(\neg A \wedge \neg B)$

b) $A \wedge (A \rightarrow B) \vdash_{\tau} B$

c) $A \rightarrow (B \rightarrow C) \vdash_{\tau} (A \rightarrow B) \rightarrow (A \rightarrow C)$

Exercise 3.4 [Gentzen Calculus]

Prove:

a) $\neg(p \rightarrow q) \vdash_G q \rightarrow p$

b) $\vdash_G (p \wedge q) \rightarrow p \vee r$

c) $s \wedge r, r \rightarrow \neg(p \wedge q) \vdash_G \neg p, \neg q$

Delivery: until 24.5.2013 12:00 Uhr into the box next to room 34/401.4