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Games with perfect information
Exercise sheet 11

TU Braunschweig
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Out: June 23 Due: June 30

Submit your solutions until Friday, June 30, 14:00, in the box next to office 343.

Exercise 1

Let $\mathcal{G}$ be a zero-sum game. Prove that if $\mathcal{G}$ has a value, it has a unique value.

Use the definitions in your proof, do not use $\nu_{\bigcirc} > \nu_{\square}$ or the minmax theorem.

Exercise 2

Let $\mathcal{G}$ be a length-k payoff game on a finite game arena $G = (V_{\square} \cup V_{\bigcirc}, R)$ for some initial position $x_0 \in V$. We assume that there is a weight-function $w \colon R \to \mathbb{Z}$ assigning each arc is weight as an integer. The value of the payoff function φ of a play $p = r_0 r_1 r_2 \ldots r_n$ of length $n \leq k$ is defined as follows:

$$\varphi(p) = \sum_{i=0}^{n} w(r_i) \, .$$

Present a recursive algorithm determining the value ν of such a game.

Hint: For each position y and each number $n \leq k$, define $\nu_k(y)$ as the value achieved in the game where we see y as the initial position and n as the bound on the length of plays. Show how to compute these values.

Exercise 3

Let $G = (V_{\square} \cup V_{\bigcirc})$ be a finite, deadlock-free game arena, $x_0 \in \bigcirc$ an initial position and $w \colon R \to \mathbb{R}$ a weight function. We assume that G is bipartite, i.e. the players alternately take turns.

We define a finite game $\mathcal{G}^{fin}$ as follows: The players pick moves, starting from position x_0.

A **repetition** is a move $r_m = (x, y) \in V_{\star} \times V_{\overline{\star}}$ such that there is an $\ell < m$ with $r_\ell = (y, z) \in V_{\overline{\star}} \times V_{\star}$.

The plays of $\mathcal{G}^{fin}$ stops on the first repetition, say for the moves $\ell < m$. Then refuter pays to prover the value

$$\varphi^{fin}(p) = \frac{1}{m - \ell + 1} \sum_{i=\ell}^{m} w(r_i) \, .$$

a) Prove that if the moves corresponding to $\ell < m$ form a repetition, then $\ell + m$ is odd.

b) Derive a bound on the length of maximal plays of $\mathcal{G}^{fin}$.

c) Prove that $\mathcal{G}^{fin}$ has a value by modeling $\mathcal{G}^{fin}$ as a zero-sum game of bounded length.