Games with perfect information
Exercise sheet 7

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Out: May 18 Due: May 26

Submit your solutions until Friday, May 26, 14:00, in the box next to office 343.

Exercise 1: Encoding winning conditions

Let $G = (V_\Box \cup V_\bigcirc, R)$ be a deadlock-free, finite game arena. Let $x, y \in V$ be two positions, $x \neq y$.

a) Present a reachability/safety game whose winning condition encodes the following property: A play is won by refuter if it visits first x, then y.

   Note: You are allowed to modify the game arena G.

b) Present a reachability/safety game whose winning condition encodes the following property: A play is won by prover if it does not visit both x and y.

c) Present a Büchi/coBüchi game whose winning condition encodes the following property: A play is won by refuter if it visits x at least once, and later visits y infinitely often.

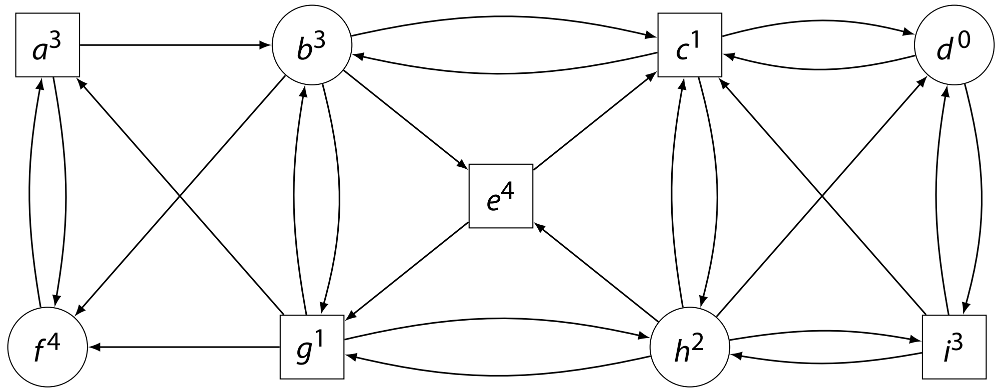
d) Present a parity game whose winning condition encodes the following property: A play is won by refuter if it either does not visit x infinitely often, or it visits both x and y infinitely often.

e) Present a parity game whose winning condition encodes the following property: A play is won by refuter if it either does not visit x infinitely often, or it visits x, but not y infinitely often.

For each part, reason briefly why your construction is correct.

Exercise 2: A parity game

Consider the parity game given by the following graph. For each vertex labeled with x^i, the letter x denotes the name of the vertex, the superscript denotes its priority $\Omega(x) = i$.



For each player, identify her winning region and present a uniform positional winning strategy. Reason briefly why the strategies are indeed winning.

Exercise 3: Proof of Lemma 8.5

Proof Part b) of Lemma 8.5 from the lecture notes:

Let X be a set of positions such that $\star \in \{\Box, \bigcirc\}$ has a positional winning strategy $s_{\star,x}$ for each $x \in X$. Then there is a positional strategy $s_\star$ that is uniformly winning from all positions $x \in X$.