Concurrency theory
Exercise sheet 2

Sebastian Muskalla, Prakash Saivasan
TU Braunschweig

Winter term 2017/18

Out: October 25 | Due: November 1

Submit your solutions until Wednesday, November 1, during the lecture. You may submit in groups up to three persons.

Exercise 1: Petri net constructions

a) Let (N, M_0, M_f) be a Petri net. Explain how to construct a Petri net (N', M'_0, M'_f) with $M'_0(p) = 0$ for all places but a single place p' with $M'_0(p') = 1$ and $M'_f(p) = 0$ for all places such that $M_f \in R\big(N, M_0\big)$ iff $M'_f \in R\big(N', M'_0\big)$.

b) Let (N, M_0, M_f) be a Petri net. Explain how to construct a Petri net (N', M'_0, M'_f) such that M_f is coverable from M_0 in N iff M'_f is reachable from M'_0 in N'.

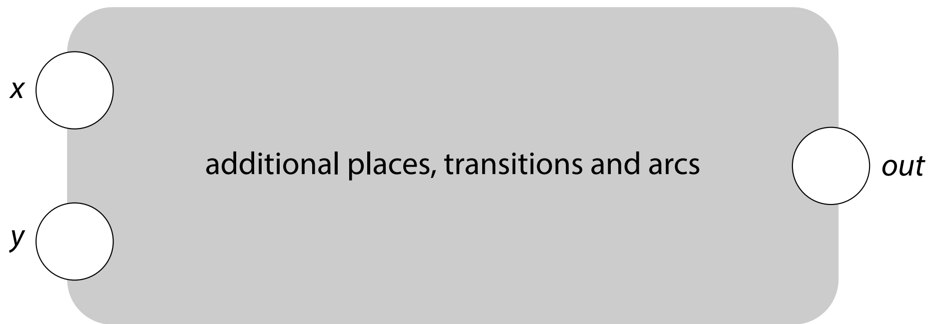
c) Construct a Petri net N with only 3 places, a marking M_0 and markings $M_{c \wedge r}, M_{\neg c \wedge \neg r}$ and $M_{c \wedge \neg r}$ such that

- $M_{c \wedge r}$ is reachable and coverable from M_0,
- $M_{\neg c \wedge \neg r}$ is neither reachable nor coverable, and
- $M_{c \wedge \neg r}$ is coverable, but not reachable.

In each part of this exercise, argue briefly that your construction is correct.

Exercise 2: Addition and multiplication

Consider the (incomplete) Petri net containing places x, y and *out* depicted below.



a) Add places and transitions to the net such that any computation of the net starting in $M_0(x) = m, M_0(y) = n, M_0(out) = 0$ terminates in a marking M_f with $M_f(out) = m + n$.

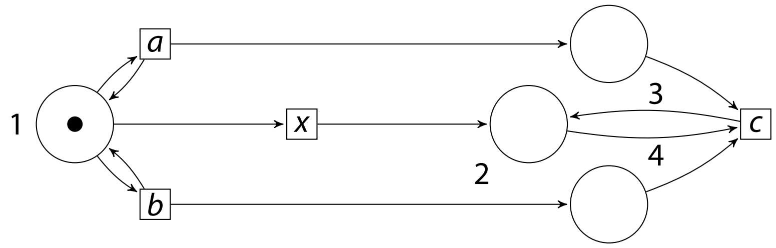
   (*Terminating* means that no transition is enabled anymore.)

b) Add places and transitions to the net such that any computation of the net starting in $M_0(x) = m, M_0(y) = n, M_0(out) = 0$ terminates in a marking M_f with $M_f(out) \in \big\{0, \ldots, m \cdot n\big\}$.

In each part of this exercise, argue briefly that your construction is correct.

Exercise 3: Rackoff's bound

Consider the Petri net $N = (\{1,2,3,4\}, \{a,b,c,x\}, \mathfrak{i}, \mathfrak{o})$ with multiplicities as depicted below. The initial marking of interest is $M_0 = (1,0,0,0)^T$ and the final marking is $M_f = (1,0,10,100)^T$.



Compute the values $m(3, M_0)$ and $f(3)$ and argue why they are correct.

Exercise 4: Counter programs

You may use additional counter variables to solve these problems. In each part of this exercise, you may use the previous parts as subroutines.

Let n be some fixed number.

a) Present a counter program $\texttt{Set}_n(x_j)$ that sets the value of counter variable x_j to n.

b) Present a counter program $\texttt{Double}(x_j)$ that doubles the current value of counter variable x_j.

c) Present a counter program $\texttt{Power}_n(x_j)$ that sets the value of counter variable x_j to 2^n.

d) Present a counter program $\texttt{Square}(x_j)$ that squares the value of counter variable x_j, i.e. the new value is v^2, where v is the old value.

In each part of this exercise, argue briefly that your program is correct.