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Exercises to the lecture
Complexity Theory
Sheet 7

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Thomas Haas Delivery until 07.02.2022 at 15:00

Exercise 7.1 (Triangle Packing)

We want to construct a randomized algorithm for the following problem:

Triangle Packing (TP)	
Input:	An undirected graph G, and a bound $k \in \mathbb{N}$.
Parameter:	k.
Question:	Does G contain k triangles that do not share any vertex?

a) Use Color Coding (with $3k$ colors) to reduce the problem to a colorful version.

b) Find a dynamic programming algorithm for the problem in a), running in $\mathcal{O}(2^{3k} \cdot n^3)$.

c) State an algorithm for TP. What is the runtime if we want constant error probability?

d) Derandomize your algorithm using hash functions. What is the resulting runtime?

Exercise 7.2 (Tree Subgraph Isomorphism)

Consider the following problem:

Tree Subgraph Isomorphism (TSI)	
Input:	An undirected graph G, and a tree T with k nodes.
Parameter:	k.
Question:	Is there a subgraph of G that is isomorphic to T?

Develop a randomized algorithm for TSI with constant error probability. Base the algorithm on Color Coding.

Hint: Proceed like above. First, reduce the problem to a colorful formulation. Then solve the colorful problem via dynamic programming. To this end, root the tree at some node r and let T_x denote the subtree of T, rooted in a node x. Moreover, let S denote a set of colors and let V_i be those vertices of G that are colored by i. Use a function $\mathit{Tree}[S, T_x, v]$, which returns true if and only if $|S| = |V(T_x)| - 1$, the color of v is not in S and there is a colorful subgraph of $G[\bigcup_{i \in S} V_i \cup \{v\}]$ which is isomorphic to T_x such that v corresponds to x in the isomorphism. Find a suitable recurrence and deduce an algorithm for TSI.

We wish you a merry Christmas and a happy new year 2018. Enjoy your vacation!

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