Automata constructions

UNION
CONCATENATION
PROJECTION
ITERATION
INTERSECTION

POWERSET

On all automata models we considered (when applicable)

Decision Procedures

$\mathcal{L}(A) = \emptyset$ EMPTINESS

$\mathcal{L}(A) = \Sigma^*$ UNIVERSALITY

$\mathcal{L}(A) \subseteq \mathcal{L}(B)$ INCLUSION (via $\mathcal{L}(A) \cap \mathcal{L}(\bar{B}) = \emptyset$)

$\mathcal{L}(A) = \mathcal{L}(B)$ EQUIVALENCE

$A \models \varphi$ as $\mathcal{L}(A) \subseteq \mathcal{L}(\varphi)$

Complementation

- via determinisation
- via equivalence relation
- via negation elimination/quant elim.
- via characterisation of winning strategies of parity games

TRIANGLES (Expressivity)

NFA — WMSO
UFO
REG
SF

$\boxed{\text{Triangle + Decision proc. on NFA} \Rightarrow \text{WMSO sat is decidable}}$

REG $\subseteq$ NFA by closure properties of NFA
NFA $\subseteq$ REG by Arden's Lemma

COMPLEMENTATION via DETERMINISATION
NFA = DFA
by Powerset constr.

NFA definable $\subseteq$ MSO definable   Encoding runs with $\exists Q_0 \dots Q_m$

MSO $\subseteq$ NFA $\neg\varphi \rightsquigarrow$ via complementation
$\exists x, \exists X$ via Σ_V alphabet extension and projection

WMSO = $\exists$WMSO

For proving FO $\subset$ WMSO we need powerful theory of EF Games

FO = SF via finiteness of classes of $\equiv_{k,m}$ NO IN EXAM

EF GAMES Key ideas:

- quantifier depth
- Equivalence $(S_v, \vec{s}) \equiv_{k,m} (S_w, \vec{t})$
- EF-Theorem:

$\boxed{\text{Duplic. wins } G_k(S_v\vec{s}, S_w\vec{t})}$ iff $\boxed{S_v\vec{s} \equiv_{k,m} S_w\vec{t}}$

Know how to use it!
e.g. to prove $(aa)^*$ not FO[<]-defin.

find for every $k \in \mathbb{N}$, a pair of str.
v_k w_k s.t. Dup wins
and $v_k \in L$ but $w_k \notin L$
$\Rightarrow$ then by EF-theorem, L not FO[<]-defin.

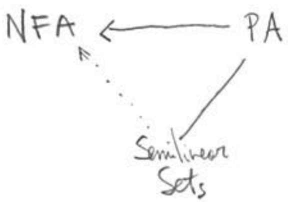
Remember key idea behind proof EF-theorem:
formula char. winning cond of D (not the details)

Logics

- MSO, FO } over words, ω-words, trees, infinite trees
  Domain = positions
  Predicates = <, succ, P_a
- Presburger Arithmetic FO over $\mathbb{N}$ + <
- LTL

Games

- EF-Games
- Parity games



PA is decidable
- by NFA
- by Quant. Elimination

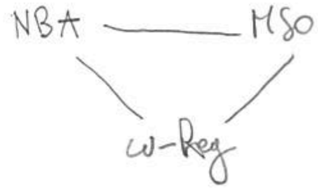
PA ≡ Semi Linear Sets
definable

- Resolve negation and $\exists x$ via Quant Elim
- Show closure <u>under $\cap$</u> (and $\cup$)

CONCEPT of PARIKH IMAGES and relation with Semilinear Sets

$\Psi(CFG)$ is semilinear

<u>Application</u>: Rev. Bounded NOT IN THE EXAM



ω-Reg = NBA by closure props.
- key constr. NBA $\cap$ by using flags

DBA less expr. than NBA

<u>BIG RESULT</u>: COMPLEMENTATION
- via Finite quotient of Σ^ω as seen by A

<u>KEY Tool</u> Ramsay's Theorem

<u>Algorithmic approach</u> with boxes → NOT IN EXAM

CONSEQUENCE: MSO over ω-words is decidable

Emptyness: lasso finding

LTL ($\equiv$ FO on ω-words)

LTL → NBA via Hintikka Sets

AUTOMATA AS DATA STRUCTURES (2)

- NFA for Solution Spaces of Presb. Arith.
- Parikh images as Semilinear Sets
- Model Checking PushDown Systems:
  - pre* construction
  - Model checking LTL using BPDS [via "advanced" lasso finding via pre*]

BUTA
TDTA — (MSO over finite trees)

def

RegTree

Key facts: D TDTA less expr. than TDTA

TDTA ≡ BUTA

BUTA ≡ DBUTA ⇝ COMPLEMENTATION via Determinisation

Decision proc: EMPTINESS (extension of reach. for NFA)

PTA — MSO over inf trees

def

ωRegTree

DPTA less expr. than PTA

Deep result: COMPLEMENTATION of PTA via Games and Determinacy

EMPTINESS via solving Parity Game

CONSEQUENCES: MSO is decidable over inf trees (S2S)

PARITY GAMES:

- Definition
- ATTRACTOR CONSTR.
- Determinacy result
- Solver exists and has complexity NP ∩ co-NP
- Parity Automata

GOOD LUCK!

And remember:

"Imagination is more important than knowledge"

BUT ONLY AFTER PASSING THE EXAM !!!